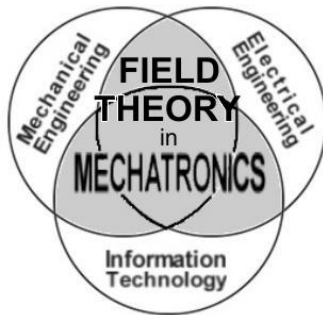


# ELECTROMAGNETIC FIELD THEORY

## Electrodynamics and its analogies in physics based on extended Maxwell's equations for industrial applications in mechatronics



Guest Lectures at the **Technical University Opole (Poland)**,  
Swiss German University Jakarta (Asia) and other universities for the faculties  
**Mechatronics + Electrical Engineering and Information Technology**

provided by  
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These lectures at University of Applied Sciences Koblenz (Germany) and Technical University Opole (Poland) are based on contributions at REM conference Research and Education in Mechatronics [11], own publications and books about field theory and industrial mechatronics [1, 3], lectures at intl. universities i.e. SGU Asia [13], results of an advised Master Thesis at SGU about robotics [2], research and developments for motor car production [9] and own web sites about Electromagnetic Field Theory using Heaviside's streamlined re-design of Maxwell's equations and extensions [10].

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## ABSTRACT

A high aim of mechatronics is the integration of mechanical engineering, electrical engineering and information technology including computer-aided simulations for all different disciplines in this complex field of application. But very often an optimized or new mechatronic design cannot be developed if engineers are not experienced in electrodynamics based on extended Maxwell's equations including movable bodies [1-14]. Therefore this publication shall demonstrate:

*First*, how easily engineers can understand and derive the complete Maxwell's equations in differential and integral form with additional views from mnemonics and entropy, too. The usage of Mind Maps and Memo Maps in physics is helpful for a good understanding.

*Second*, based on Maxwell's equations the analogies between magnetostatics, electrostatics, and electric current flow based on concentrated field elements are useful tools for analytical approximations in all applications of interdisciplinary mechatronics.

*Third*, the mighty capabilities using "Unit Check" methods will show how we can derive a huge range of famous equations in physics primarily electrodynamics (extended Maxwell's equations up to formulas in quantum mechanics, Einstein's special relativity and more) without using any additional literature. This method is a good exercise to train our brain.

*Fourth*, the structure identity of the complete eddy current equation with respect to other disciplines in physics (i.e. hydrodynamics, thermodynamics and elastomechanics) opens the door for an interdisciplinary development and optimization of new mechatronic systems.

An important aim of this publication is to provide a helpful re-formulated set of extended Maxwell equations and a variety of methods for mechatronic engineers and students with respect to understanding, approximation, integration and solution of different physical disciplines. Actual computer-aided applications in the broad field of industrial motor car production, robot gripper design, anti-vibration systems with software controlled actuators and sensors and computer hard disc drives will show the high efficiency and central position of extended Maxwell's equations in mechatronics.

## 1 INTRODUCTION

The often recognized problem in mechatronics is a lack of experience in applying electrodynamic knowledge. Therefore a compact introduction in an extended Maxwell's field theory with interdisciplinary applications shall introduce a valuable key for all "Mechatronicists". All the described industrial developments were primarily based on electrodynamics, using innovative ideas, Maxwell's equations and both software controlling and computer-aided simulation. But the focus of this publication is primarily on the advantage and necessity of electrodynamics inside mechatronics.

## 2 ELECTRODYNAMICS AS A CENTRAL PART IN MECHATRONICS

The fact that electrodynamics is a central part in mechatronics will be shown by different views of Maxwell's equations and interdisciplinary evaluations.

### 2.1 Electrodynamics based on Maxwell's equations

One of the most famous formulations in physics is the set of Maxwell's equations. In the next lines some basic and extended equations will be shown or re-formulated.

#### 2.1.1 Basic Maxwell's equations and constitutive relations

A compact overview of basic Maxwell's equations in differential and integral formulation with (nonlinear) constitutive relations is presented in this section.

**Maxwell's equations in differential form**

The basic set of Maxwell's equations (1) - (4) can be written in differential form:

$$\nabla \times \mathbf{H} = \mathbf{J} + \dot{\mathbf{D}} \quad (1)$$

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}} \quad (2)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (4)$$

**Fig 1. Set of Maxwell's equations with equivalent mnemonics "Maxwell's Hand" [1], [10]**

Eq. (1) in Fig. 1 is Ampere-Maxwell's Law and eq. (2) Faraday-Lorentz' Law, both of which are called field equations. Eq. (3) is electric Gauss' Law and eq. (4) magnetic Gauss' Law, both are called source equations for Maxwell's field theory.  $\mathbf{B}$  is magnetic flux density in Vs/m<sup>2</sup>,  $\mathbf{H}$  the magnetic field strength in A/m,  $\mathbf{D}$  is displacement or electric flux density in As/m<sup>2</sup>,  $\mathbf{E}$  the electric field strength in V/m,  $\mathbf{J}$  is the electric current density in A/m<sup>2</sup>,  $\rho$  is the electric volume charge density in As/m<sup>3</sup>,  $Q$  is electric charge in As, and  $\nabla$  is the Nabla-Operator for vector analytical operations. For all bodies in rest, the dot (•) over  $\mathbf{D}$  and  $\mathbf{B}$  means partial derivatives of these characteristics with respect to time (here  $d/dt = \partial/\partial t$ ). Simple mnemonics are shown in Fig.1.

**Maxwell's equations in integral form**

Using the known vector analysis laws by Stokes and Gauss we get from eq. (1) - (4):

$$\oint \mathbf{H} d\mathbf{l} = \iint \mathbf{J} d\mathbf{s} + \iint d\mathbf{D}/dt \cdot d\mathbf{s} \quad (1a)$$

$$\oint \mathbf{E} d\mathbf{l} = - \iint d\mathbf{B}/dt \cdot d\mathbf{s} \quad (2a)$$

$$\iint \mathbf{D} d\mathbf{s} = \iiint \rho d\mathbf{v} = Q \quad (3a)$$

$$\iint \mathbf{B} d\mathbf{s} = 0 \quad (4a)$$

Because of primarily using the superior magnetic vector potential  $\mathbf{A}$  shown in later equations, we introduce letter  $\mathbf{s}$  (=surface) for area,  $\mathbf{l}$  is the length and  $\mathbf{v}$  is the volume. If we don't consider moving bodies, the terms  $d/dt$  are partial derivatives  $\partial/\partial t$ .

**Constitutive relations**

The constitutive relations between the classical field terms  $\mathbf{D}$ ,  $\mathbf{E}$ ,  $\mathbf{B}$ ,  $\mathbf{H}$  and  $\mathbf{J}$ , also including both polarisations and external current sources, are defined by eq. (5) - (7):

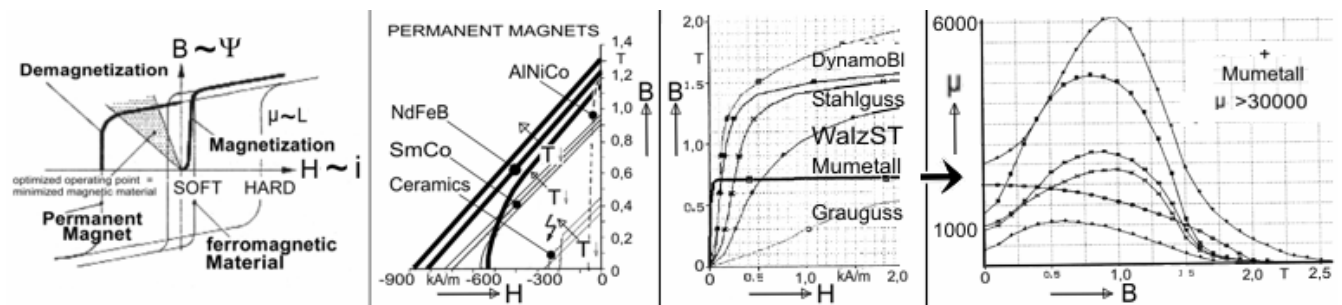
$$\mathbf{D} = [\epsilon] \mathbf{E} + \mathbf{P} \quad (5)$$

$$\mathbf{B} = [\mu] \mathbf{H} + \mathbf{B}_p \quad (6)$$

$$\mathbf{J} = [\gamma] \mathbf{E} + \mathbf{J}_e \quad (7)$$

Eq. (6) with details:  $\mathbf{B} = \mu \mathbf{H} + \mathbf{B}_p = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}_e + \mu_0 \mathbf{M}_p = \mu_0 (\mathbf{H} + \mathbf{M}_e) + \mu_0 \mathbf{M}_p \quad (6a)$

In eq. (6a)  $\mathbf{B}_p$  is the magnetic polarization and  $\mathbf{M}_p$  is the magnetization in permanent magnets,  $\mathbf{M}_e$  is the magnetization in magnetic iron caused by an external field (index "e"), considering magnetic iron without permanent magnets  $\mathbf{B}_p = 0$ , without iron  $\mathbf{M}_e = 0$ , too [14]. The material property  $\mu = \mu_0 \cdot \mu_r$  is the permeability in ferromagnetic materials,  $\epsilon = \epsilon_0 \cdot \epsilon_r$  is the permittivity in dielectric materials and  $\gamma$  is the electrical conductivity.  $\mathbf{P}$  is the electric polarization,  $\mathbf{J}_e$  are all possible external current sources. In most of industrial applications magnetic material properties, primarily permeability, show non-linear characteristics, ref. Fig. 2. [ $\mu_0 = 4\pi \cdot 10^{-7}$  Vs/Am =  $1/(c^2 \cdot \epsilon_0)$ ]



**Fig. 2 Graphical constitutive relations of permanent magnets + ferromagnetic materials [1], [11]**

### 2.1.2 Extended Maxwell's equations considering moved bodies

Following four re-formulated Maxwell equations (1b) - (4b) can be used for all advanced calculations and computations in electrodynamics (with fields and waves), including constitutive relations [ref. to eq. (5)-(7)] and arbitrary movements of bodies (or particles) with speed  $\mathbf{v}$ . The basis of these extensions is the relativity relation  $(\mathbf{v} \cdot \nabla) \mathbf{A} = d\mathbf{A}/dt - \partial \mathbf{A} / \partial t$  (ref. to Einstein's Relativity Theory [12], Helmholtz' theorems for moved objects [1], [10], Sommerfeld's electrodynamics [7]), where  $\mathbf{A}$  may be any vector, scalar or tensor. Furthermore these equations are the central basis for understanding interdisciplinary physics, especially structure identical formulations in i.e. hydrodynamics, diffusion, thermodynamics etc compared with directly derivable eddy current equation. Material properties of  $[\mu]$ ,  $[\varepsilon]$  and  $[\gamma]$  in brackets shall be a reminder, that they are often non-linear and additionally tensors.

1. extended Maxwell's equation <b>Ampere-Maxwell's Law</b>	$\nabla \times \mathbf{H}' = + \left( \frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \right) \cdot \mathbf{D} + \mathbf{J}$	(1b)
2. extended Maxwell's equation <b>Faraday-Lorentz' Law</b>	$\nabla \times \mathbf{E}' = - \left( \frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \right) \cdot \mathbf{B}$	(2b)
3. extended Maxwell's equation <b>electric Gauss' Law</b>	$\nabla \cdot \mathbf{D}' = \rho'$	(3b)
4. extended Maxwell's equation <b>magnetic Gauss' Law</b>	$\nabla \cdot \mathbf{B}' = 0'$	(4b)
→ using $\mathbf{B}, \mathbf{H}, \mathbf{D}, \mathbf{E}$ etc area based vector analysis (8a)	$(\mathbf{v} \cdot \nabla) \cdot \mathbf{B} = -\text{curl}(\mathbf{v} \times \mathbf{B}) + \mathbf{v} \cdot \text{div} \mathbf{B} - \mathbf{B} \cdot \text{div} \mathbf{v} + (\mathbf{B} \cdot \text{grad}) \mathbf{v}$	(8a)
→ using $\mathbf{A}$ ( with $\mathbf{B} = \text{curl} \mathbf{A}$ ) line based vector analysis (8b)	$(\mathbf{v} \cdot \nabla) \cdot \mathbf{A} = -\mathbf{v} \times \text{curl} \mathbf{A} + \text{grad}(\mathbf{v} \cdot \mathbf{A}) - (\mathbf{A} \cdot \text{grad}) \cdot \mathbf{v} - \mathbf{A} \times \text{curl} \mathbf{v}$	(8b)

Fig 3. Extended Maxwell's equations for moved bodies and basics in vector analysis [1], [10]

The transformation equations in *general* formulation are:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} + \dots \text{further terms} \quad \mathbf{H}' = \mathbf{H} - \mathbf{v} \times \mathbf{D} + \dots \text{further terms} \rightarrow \text{refer to eq. (8)}$$

The additional field entities i.e.  $\mathbf{v} \times \mathbf{B}$  and  $\mathbf{v} \times \mathbf{D}$  - caused by moved bodies - are only 1 of 4 possible terms. The transformation equations in *simplified* formulation are therefore

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} \quad (2c) \quad \text{and} \quad \mathbf{H}' = \mathbf{H} - \mathbf{v} \times \mathbf{D} \quad (1c)$$

and well known as basic Lorentz' Transformation. Using only this special transformation the transformed current caused by moved body (i.e. conductor) is  $\mathbf{J}' = \mathbf{J} - \mathbf{v} \bullet \rho$ . (1e)

Following three examples shall deepen the background about influences of transformations:

**1. EXAMPLE** for evaluation with magnetic flux density terms. Derivation of Faraday-Lorentz' Law using equation (8a): Assuming special conditions/restrictions (in literature often not mentioned) i.e. incompressible materials  $\text{div} \mathbf{v} = 0$ , space independent constant movements  $(\mathbf{B} \cdot \text{grad}) \mathbf{v} = 0$  and in magnetic fields directly from magnetic Gauss' law always  $\text{div} \mathbf{B} = 0$  the remaining term on the right side in eq. (8a) yields  $\text{rot}(\mathbf{B} \times \mathbf{v}) = -\text{rot}(\mathbf{v} \times \mathbf{B}) = -\text{curl}(\mathbf{v} \times \mathbf{B})$ . Inserting this result in Faraday's Law we can simply derive the extended 2. Maxwell's equation for moved bodies: differential Faraday - Lorentz' - Law

$$\text{curl} \mathbf{E}' = -d\mathbf{B}/dt = -\partial \mathbf{B} / \partial t + \text{curl}(\mathbf{v} \times \mathbf{B}) \quad (2d)$$

using equation (8b) with same conditions mentioned above we get eq. (2d) with  $\nabla \times \mathbf{A} = \text{Nabla} \times \mathbf{A} = \text{curl} \mathbf{A} = \mathbf{B}$ . The first term on the right side of this equation (2d) was proved by Faraday, the second one by Lorentz. NOTE: using this vector analytical formulation we get the Lorentz-Term  $\mathbf{E} = \mathbf{v} \times \mathbf{B}$  automatically! The famous Lorentz law is therefore a (very important) vector identity ... but not really a separate physical law.

**2. EXAMPLE** for evaluation with electric flux density terms. Derivation of Ampere-Maxwell's Law using equation (8a): Assuming special conditions/restrictions as in the 1. Example (i.e. incompressible materials  $\text{div} \mathbf{v} = 0$ , space independent constant movements  $(\mathbf{D} \cdot \text{grad}) \mathbf{v} = 0$  and in electric fields directly from electric Gauss' law  $\text{div} \mathbf{D} = \nabla \bullet \mathbf{D}$  the remaining term on the right side in eq. (8a) yields for the cross product  $\text{rot}(\mathbf{D} \times \mathbf{v}) = -\text{rot}(\mathbf{v} \times \mathbf{D}) = -\text{curl}(\mathbf{v} \times \mathbf{D})$  and in opposite to the Faraday-Lorentz' Law in eq. (2d) for the Ampere-Maxwell's Law in equation (1d) an additional term. Considering simplified conditions like non-relativistic, linear and constant Movements yields eq. (3). But generally  $\mathbf{v} \bullet \text{div} \mathbf{D}' = \mathbf{v} \bullet \rho'$  is valid, ref. to eq. (3b), [7], [10].

Inserting these results in Ampere-Maxwell's Law eq. (1a) we can derive the extended 1. Maxwell's equation for moved bodies or particles with following expressions:

$$\text{curl } \mathbf{H}' = \mathbf{J} + d \mathbf{D} / dt = \mathbf{J} + \partial \mathbf{D} / \partial t + \mathbf{v} \cdot \rho - \text{curl} (\mathbf{v} \times \mathbf{D}) \quad (1d)$$

The first term on the right side of this equation (1d) was proved by Ampere, the third term by Rowland, the second term by Hertz (suggested and introduced by Maxwell), the fourth term by Roentgen. NOTE: using this vector analytical formulation we get the "dualism" of the Lorentz-Term  $\mathbf{H} = -\mathbf{v} \times \mathbf{D}$  automatically! The Rowland and Roentgen terms are therefore (important) vector identities ... but not really separate physical laws.

**3. EXAMPLE:** Proof of extended 1. + 2. Maxwell's equations using famous HELMHOLTZ' formula. Helmholtz derived for any arbitrary vector flux  $\mathbf{X}$  in physics (i.e. hydrodynamics) through a moved ( $\mathbf{v}$ ) and simultaneously deformable area element in his curl laws - as a subset of (8a) - following formula :

$$d \mathbf{X} / dt = \partial \mathbf{X} / \partial t + \text{curl} (\mathbf{X} \times \mathbf{v}) + \mathbf{v} \text{ div } \mathbf{X} \quad (*)$$

Inserting this Helmholtz' formula (\*) in the Maxwell equations (1a) and (2a) - prerequisiting both the same above mentioned conditions and  $\mathbf{X} = \mathbf{B}$  alternatively  $\mathbf{X} = \mathbf{D}$  - we immediately get the extended Maxwell's equations (1b) and (2b) in the 1. and 2. example! NOTE: using (\*) the extended Maxwell's equations are derivable without any knowledge in vector analysis. The Helmholtz' formula is ingenious and the basis for Lorentz, Minkowski and Einstein, too. Helmholtz derived his formula visualizing - like a "mnemonics artist" - moved and deformable geometric elements. But nevertheless Helmholtz' formula  $d \mathbf{X} / dt$  neglects the LAST term, here  $(\mathbf{X} \cdot \nabla) \mathbf{v}$  inside  $(\mathbf{v} \cdot \nabla) \mathbf{X}$  (i.e. additional rotations), refer to (8a) and (8b) !

### 2.1.3 Extended Maxwell's equations in 4-dimensional formulation

Another compact expression of Maxwell's equations (i.e. in vacuum without materials and no movable bodies) can be derived, using 4-dimensional expressions [7], [1]:

1. space-time operator  $\square (x, y, z, i \cdot c \cdot t)$  with d'Alembert  $\square \equiv \Delta - 1/c^2 \cdot \partial^2 / \partial t^2 = \sum_{i=1}^4 \partial^2 / \partial x_i^2$
2.  $\mathbf{A}$ - $\varphi$ -Potential  $\Omega (A_x, A_y, A_z, i \cdot \varphi / c)$  with  $c = 1/\sqrt{(\epsilon_0 \cdot \mu_0)}$ ,  $i = \sqrt{-1}$  and
3. current densities  $\Gamma (J_x, J_y, J_z, i \cdot \rho \cdot c)$  respectively  $\mathbf{J}' = \mathbf{v} \cdot \rho$  with condensed results:

$$\text{a) } \square \Omega = -\mu_0 \cdot \Gamma, \quad \text{b) } \nabla \cdot \Omega = 0, \quad \text{c) } \nabla \cdot \Gamma = 0, \quad \text{d) } \mathbf{F} = \mu_0 \cdot \mathbf{G} = \nabla \times \Omega \quad (9)$$

where  $\mathbf{F} (\mathbf{B}, -i \cdot \mathbf{E}/c)$  and  $\mathbf{G} (\mathbf{H}, -i \cdot c \mathbf{D})$  define the electromagnetic Maxwell field tensors.

### 2.1.4 Extended Maxwell's equations in quantum electrodynamics

Quantum electrodynamics is a complex interdisciplinary field, but normally not used daily by practical mechatronics engineers. On the other side many phenomena (duality of wave and particle, tunnel diode, special superconductivity up to quantum computers etc) are important and must be handled with a background of this superior theory based on the integration of electrodynamics, quantum mechanics and (for relativistic processes) relativity theory [1]. As a compromise only the resulting extended Maxwell equations in quantum electrodynamics will be shown in (1f) - (4f).

$$\nabla \times \mathbf{H} = \mathbf{J} + \overset{\bullet}{\mathbf{D}} - \kappa^2 \cdot \mathbf{A} / \mu_0 \quad (1f) \quad \nabla \times \mathbf{E} = -\overset{\bullet}{\mathbf{B}} \quad (2f) \quad \nabla \cdot \mathbf{D} = \rho - \kappa^2 \cdot \varphi \cdot \epsilon_0 \quad (3f) \quad \nabla \cdot \mathbf{B} = 0 \quad (4f)$$

These extended Maxwell's equations, which called Proca's equations, additionally describe special phenomena in quantum electrodynamics [5], [1].

These further quantum terms consist of classical magnetic vector potential  $\mathbf{A}$ , respectively electrical scalar potential  $\varphi$ , the special term  $\kappa^2 = (m_0 \cdot c / \hbar)^2$  and material properties in vacuum both permeability  $\mu_0$  and permittivity  $\epsilon_0$ .

The term  $\kappa^2$  is famous in quantum mechanics, because  $\kappa$  is Compton's frequency divided by speed of light  $c$  or Einstein's energy in view of quantum mechanics. The mass in rest is  $m_0$ , the universal Planck's constant in quantum mechanics is  $\hbar$  ( $= h / 2\pi \approx 1 \cdot 10^{-34}$  J s).

## 2.2 Interdisciplinary evaluation of Maxwell's equations

From Maxwell's equations we can directly derive all central relations for electromagnetic waves and fields, eddy current equations, structure identities inside electrodynamics and with other physical disciplines, too. Ref. to all possible derivations in chap. 2.2.4)

### 2.2.1 Electromagnetic Field and Wave equations

Electrodynamics as 1 compact equation including polarizations and movable bodies:

$$\text{curl} \frac{1}{\mu} \text{curl} \mathbf{A} = \mathbf{J}_e + \text{curl} \frac{1}{\mu} \mathbf{M}_p + \frac{\partial \mathbf{P}}{\partial t} + \mathbf{v} \cdot \rho + \left( \gamma + \varepsilon \frac{\partial}{\partial t} \right) \cdot \left[ -\text{grad} \varphi - \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v} \times \text{curl} \mathbf{A} \right] \quad (10)$$

Choice of gauges in electrodynamics is important for evaluation of fields and waves, because potentials  $\mathbf{A}$  and  $\varphi$  are not unique ( $\Psi$  scalar magnetic potential), eq.(10).

$$\mathbf{A} = \mathbf{A}^* - \nabla \Psi, \quad \varphi = \varphi^* + \partial \Psi / \partial t \quad (10a+b) \quad \Delta \mathbf{A} - \mu \varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} - \mu \gamma \frac{\partial \mathbf{A}}{\partial t} = \nabla \cdot \left[ \nabla \mathbf{A} + \mu \varepsilon \frac{\partial \varphi}{\partial t} + \mu \gamma \varphi \right] \quad (11)$$

The most used gauges are the complete Lorentz gauge  $[\dots]=0$ , eq. (11) and reduced Lorentz gauge  $\nabla \mathbf{A} = -\mu \varepsilon \cdot \partial \varphi / \partial t$  for waves and Coulomb gauge  $\nabla \mathbf{A} = 0$  for eddy current and static applications. Wave equations from eq. (11) using eq. (3) and polarizations:

$$\Delta \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \left( \mathbf{J} + \nabla \times \mathbf{M}_p + \frac{\partial \mathbf{P}}{\partial t} \right) \quad (12a) \quad \Delta \varphi_s - \frac{1}{c^2} \frac{\partial^2 \varphi_s}{\partial t^2} = -\frac{1}{\varepsilon_0} (\rho - \nabla \cdot \mathbf{P}) \quad (12b)$$

Wave equations derived from concentrated field elements in electric circuits with resistance  $R$ , conductance  $G$ , capacitance  $C$ , inductance  $L$  (mutual inductance  $M$ ) yield the same result for voltage  $V$  and current  $I$  instead of  $\mathbf{A}$  or  $\varphi$ , as shown in eq.(11) respectively eq.(12a,b).

### 2.2.2 Eddy current equation in electrodynamics

With  $(\varepsilon \partial / \partial t) = 0$ , eq.(10) leads to interdisciplinary usage of eddy current equation(13).

$$\text{curl} \frac{1}{\mu} \text{curl} \mathbf{A} = \mathbf{J} - \gamma \cdot \text{grad} \varphi + \text{curl} \frac{1}{\mu} \mathbf{M}_p - \gamma \cdot \frac{\partial \mathbf{A}}{\partial t} + \gamma \cdot \mathbf{v} \times \text{curl} \mathbf{A} \quad (13)$$

The current density  $\mathbf{J}$  includes all further electrical excitations shown in eq. (10).

### 2.2.3 Static equations inside electrodynamics with identical structure

From Maxwell's equations we get formulations with identical structure for magnetic fields in magnetostatics, electric fields in electrostatics and electric current flow. In Fig. 4 six identical fields are sketched for different areas inside electrodynamics. The field map for only electrostatics automatically yields the results for the other shown disciplines, refer to eq.(19a, 20a). The field maps were evaluated for the centre of the applications shown neglecting the leakage fluxes i.e. of capacitor and current sheets.

"Trial and Error" field mapping proved by field numerical computations with FEM program MagnetoCAD.

Field mapping rules in Fig. 4a considering field lines and equipotential lines as perpendicular, equidistantly arranged and sketched by means of curvilinear squares.

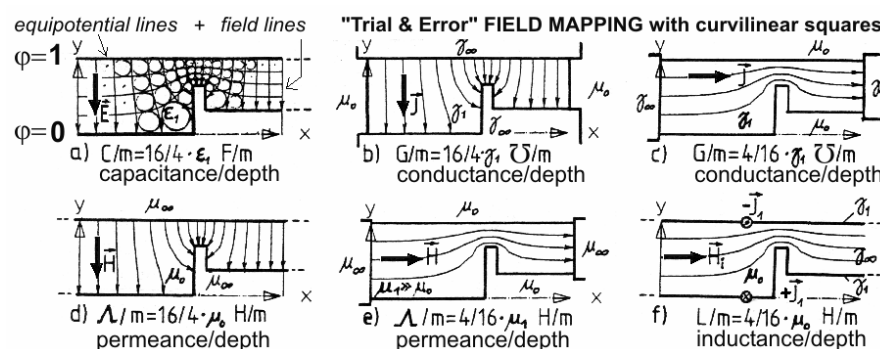


Fig 4. Application of one field map to six interdisciplinary cases inside electrodynamics [1]

## 2.2.4 All possible Mind Map derivations from variations of “Maxwell’s Hand”

All central derivations from Maxwell's equations with respect to all important phenomena inside electrodynamics are developed by the author and visualized as a new Mind Map with 10 memorable Memo Maps based on variations of Maxwell's "Right Hand Rule" and Brain Power Rules [8]. Starting from differential equations we can formulate all central equations governing electrodynamics and interdisciplinary physics. These Memo Maps are valuable mnemonics for necessary derivations, useful backgrounds and compact results. Memorizing these pictures it's easy for us to bear all derivations in mind concerning the variety of extended Maxwell's equations.

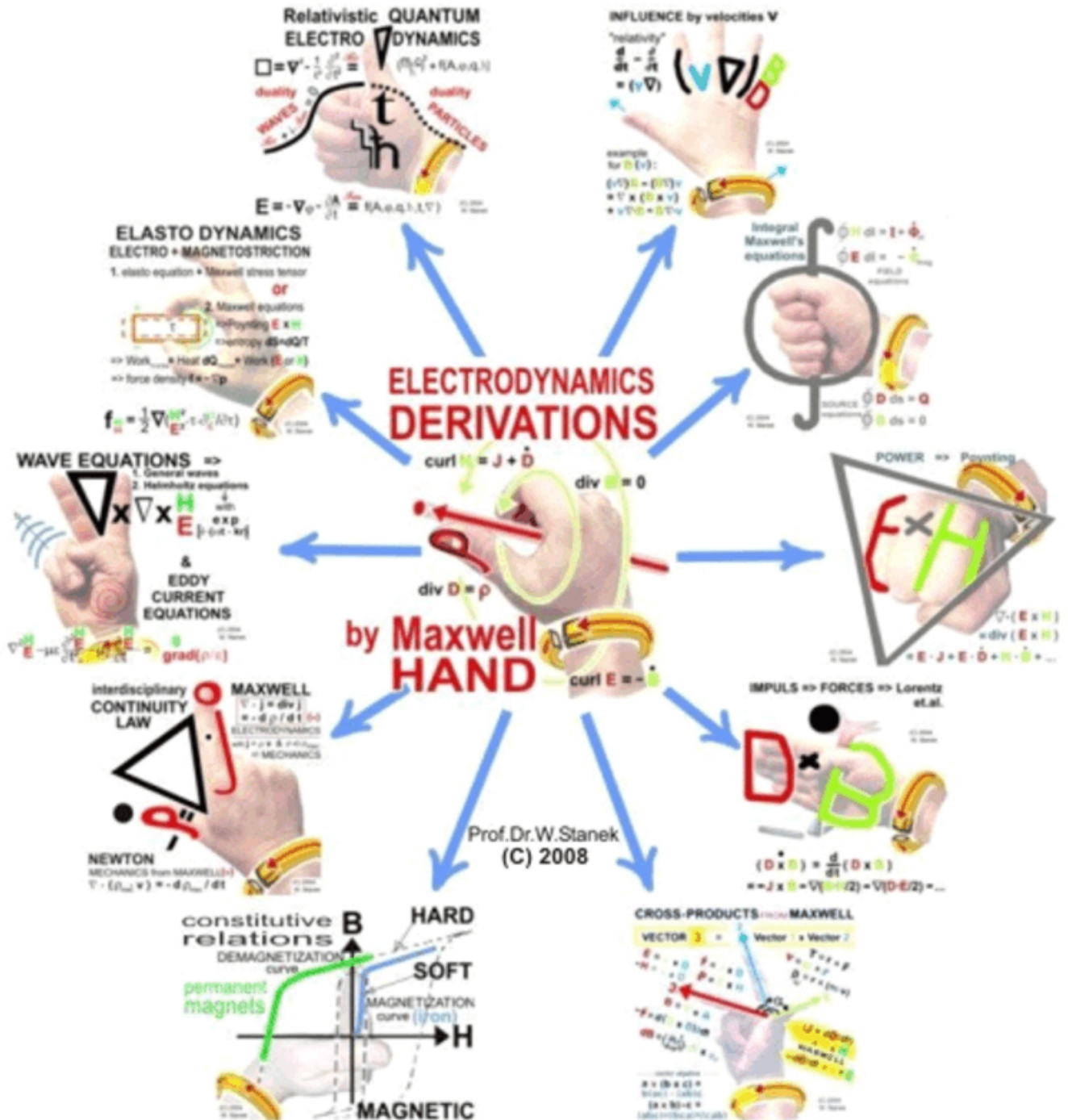


Fig. 5: Mind Map for central derivations from Maxwell's equations in electrodynamics [10]

This above shown Mind Map can be found in a special web site prepared by the author [10]. Clicking there on one of these 11 Memo Maps (including centred Maxwell's Hand = basic right hand rule) these special figures will be enlarged with further details about the mathematical procedure und the developed results for electrodynamic power, forces, energy, waves, relativity relations etc up to quantum electrodynamics. Most of all these formulas and equations can be derived using the powerful unit check method shown in the next chapter 2.2.5, too.

## 2.2.5 The mighty method in physics: Deriving equations by Unit Checks

Following questions (Q1 ... Q20) and answers (A x.y) will demonstrate and train this useful method using both sides of our brain to understand, to derive, to learn and to recall most important formulas in physics without any effort.



- Q1) What will happen with an electric Charge  $Q$  placed in an electric field  $E$ ?**  
 The scalar  $Q$  in As and vector  $E$  in V/m build the product  $Q \cdot E$ . (A1.1)  
 Equivalent unit equation: As  $\cdot$  V/m = Ws/m = Nm/m = N = Newton  $\rightarrow$  Force  $F_{el}$  (A1.2)  
 The result is Coulomb's law, an electric force  $F_{el} = Q \cdot E$  (A1.3)
- Q2) What is equivalent to a space-depending electric potential  $\varphi$  defined by gradient "grad"?**  
 This term can be written as a product of Nabla operator and electric potential  $\nabla\varphi$  (A2.1)  
 Equivalent unit equation: 1/m  $\cdot$  V = V/m electric field strength  $E$  (A2.2)  
 Regarding signs for "grad" in mathematics and  $E$  in physics the result is:  $E = - \text{grad } \varphi$  (A2.3)
- Q3) What will happen in a magnetic field  $B$  moving a particle / body with a uniform speed  $v$ ?**  
 Both vectors  $v$  in m/s and  $B$  in Vs/m<sup>2</sup> build a cross product  $v \times B$ . (A3.1)  
 Equivalent unit equation: m/s  $\times$  Vs/m<sup>2</sup> = V/m  $\rightarrow$  electric field strength  $E$  (A3.2)  
 The result is the additionally induced *electric* Lorentz' field strength  $E_L = v \times B$  (A3.3)
- Q4) What will happen in an electric field  $D$  moving a particle / body with a uniform speed  $v$ ?**  
 Both vectors  $v$  in m/s and  $D$  in As/m<sup>2</sup> build a cross product  $v \times D \rightarrow$  (ref. to Q18 !) (A4.1)  
 Equivalent unit equation: m/s  $\times$  As/m<sup>2</sup> = A/m  $\rightarrow$  magnetic field strength  $H$  (A4.2)  
 The result is the additional *magnetic* Lorentz' field strength  $H = v \times D = - D \times v$  (A4.3)  
 Applying " $\nabla$ " operator on  $H$  the result is Roentgen's current  $J_{Roe} = \nabla \times (D \times v) = \text{curl } (D \times v)$  (A4.4)
- Q5) What is equivalent to an electric charge density  $\rho$  moved with the speed  $v$ ?**  
 The scalar  $\rho$  in As/m<sup>3</sup> and vector  $v$  in m/s build the product  $\rho \cdot v$ . (A5.1)  
 Equivalent unit equation: As/m<sup>3</sup>  $\cdot$  m/s = A/m<sup>2</sup> additional electric current density  $J_{Row}$  (A5.2)  
 The result is Rowland's current density  $J_{Row} = \rho \cdot v$ . (A5.3)
- Q6) How much is the force on a current carrying conductor or moved  $\rho$  in a magnetic field  $B$ ?**  
 The physical entities  $\rho$ ,  $v$  and  $B$  build the cross product  $\rho \cdot v \times B$ . (A6.1)  
 Equivalent unit equation: As/m<sup>3</sup>  $\cdot$  m/s  $\times$  Vs/m<sup>2</sup> = Ws/m<sup>4</sup> = Nm/m<sup>4</sup> = N/m<sup>3</sup>  $\rightarrow$  force density  $f$  (A6.2)  
 The result is Lorentz' force density caused by electric currents  $f_L = J \times B$  (A6.3)
- Q7) What will happen when a magnetic flux density  $B$  is time-changing through a loop?**  
 The action  $\partial B / \partial t$  causes a reaction in a loop which must be a negatively signed vector, too. (A7.1)  
 Equivalent unit equation: 1/s  $\cdot$  Vs/m<sup>2</sup> = 1/m  $\cdot$  V/m  $\rightarrow$   $\nabla$  applied on electric field strength  $E$  (A7.2)  
 The result is Faraday's law or Maxwell's second (field) equation -  $\partial B / \partial t = \nabla \times E = \text{curl } E$  (A7.3)
- Q8) Which physical entity will be produced by an electric current density  $J$ ?**  
 All currents will produce a magnetic field strength  $H$  easily derived by following unit check:  
 Equivalent unit equation: A/m<sup>2</sup> = 1/m  $\cdot$  A/m  $\rightarrow$   $\nabla$  applied on magnetic field strength  $H$  (A8.1)  
 The result is basic Ampère's law or Maxwell's first (field) equation  $J = \nabla \times H = \text{curl } H$  (A8.2)
- Q9) Which source divergence "div" of a physical entity produces electric charge density  $\rho$ ?**  
 This relation can be written as a product of Nabla operator and electric potential  $\nabla \cdot "?" = \rho$  (A9.1)  
 Equivalent unit equation: 1/m  $\cdot$  '?' = As/m<sup>3</sup> or '?' = As/m<sup>2</sup> electric flux density  $D$  (A9.2)  
 The result is electric Gauss' law or Maxwell's third (source) equation  $\nabla \cdot D = \text{div } D = \rho$  (A9.3)
- Q10) Which magnetic source divergence "div" of a physical entity is always zero?**  
 This relation can be written as a product of Nabla operator and electric potential  $\nabla \cdot "?" = 0$  (A10.1)  
 Equivalent unit equation: 1/m  $\cdot$  '?' = Vs/m<sup>3</sup> (fictive monopole)  $\rightarrow$  magnetic flux density  $B$  (A10.2)  
 The result is magnetic Gauss' law or Maxwell's fourth (source) equation  $\nabla \cdot B = \text{div } B = 0$  (A10.3)
- Q11) Which time-changing physical entity  $X$  will produce a current density  $J$  in air (vacuum)?**  
 This relation can be written as  $\partial X / \partial t = J$ , where  $X$  is the searched unknown. (A11.1)  
 Equivalent unit equation: 1/s  $\cdot$  '?' = A/m<sup>2</sup> or '?' = As/m<sup>2</sup>  $\rightarrow$  electric flux density  $D$  (A11.2)  
 The result is Maxwell's displacement current  $J_D = \partial X / \partial t$  (= 2<sup>nd</sup> part of 1. Maxwell's equation) (A11.3)
- Q12) What is equivalent to the source "div" of a moved charge density  $\rho$  with the speed  $v$ ?**  
 Applying Helmholtz' law  $\text{div } (\text{curl } H) = 0$  on eq. (A8.2) with (A11.2) or from  $\nabla \cdot (\rho \cdot v) = "?"$  (A12.1)  
 Equivalent unit equation: 1/m  $\cdot$  (As/m<sup>3</sup>  $\cdot$  m/s) = A/m<sup>2</sup> = 1/s  $\cdot$  As/m<sup>3</sup>  $\rightarrow$   $\partial / \partial t \cdot$  charge density  $\rho$  (A12.2)  
 The result is Maxwell's continuity law in electrodynamics  $\nabla \cdot (\rho \cdot v) = \nabla \cdot J = - \partial \rho / \partial t$  (A12.3)

- Q13) What is equivalent to the source “div” of a moved mass density  $\rho_M$  with the speed  $\mathbf{v}$ ?**  
 From  $\nabla \cdot (\rho_M \cdot \mathbf{v}) = \text{“?”}$  unit equation:  $1/m \cdot (\text{kg}/\text{m}^3 \cdot \text{m}/\text{s}) = 1/\text{s} \cdot \text{kg}/\text{m}^3 \rightarrow \partial / \partial t \cdot \text{mass density } \rho_M$  (A13.1)  
 The result is Newton’s continuity law in mechanics  $\nabla \cdot (\rho_M \cdot \mathbf{v}) = \nabla \cdot \mathbf{J} = - \partial \rho_M / \partial t$  (A13.2)
- Q14) What is equivalent to the mechanical (im)pulse density  $\mathbf{p}_M = \rho_M \cdot \mathbf{v}$  in electrodynamics?**  
 Equivalent unit equation:  $\text{kg}/\text{m}^3 \cdot \text{m}/\text{s} = \text{kg} \cdot \text{m}/\text{s}^2 \cdot \text{s}/\text{m}^3 = \text{Ns}/\text{m}^3 = \text{Ws}/\text{m}^3 \cdot \text{s}/\text{m} = \text{As}/\text{m}^3 \cdot \text{Vs}/\text{m}$  (A14.1)  
 The unit Vs/m defines the magnetic vector potential  $\mathbf{A}$  with its subset  $\mathbf{B} = \text{curl } \mathbf{A} = \nabla \times \mathbf{A}$ . (A14.2)  
 Moving like Maxwell’s hand for  $\mathbf{J} = \text{curl } \mathbf{H}$  we see that 2-D fields can be calculated by 1-D! (A14.3)  
 The result is the equivalence of  $\mathbf{p}_M = \rho_M \cdot \mathbf{v}$  in mechanics and  $\mathbf{p}_{EM} = \rho \cdot \mathbf{A}$  in electrodynamics (A14.4)
- Q15) What is the relation between (im)pulse density  $\mathbf{p}_M$  or  $\mathbf{p}_{EM}$  and force density  $\mathbf{f}$ ?**  
 Equivalent unit equation from (A14.1):  $\text{Ns}/\text{m}^3 \cdot \text{“?”} = \text{N}/\text{m}^3$  or  $\text{“?”} = 1 / \text{s} \rightarrow$  time operator  $d/dt$  (A15.1)  
 a) The result in mechanics is Newton’s force law  $\mathbf{f}_M = d(\rho_M \cdot \mathbf{v}) / dt = \mathbf{v} \cdot \partial \rho_M / \partial t + \rho_M \cdot \partial \mathbf{v} / \partial t$  (A15.2)  
 b1) Result in electrodynamics is Lorentz’ force law  $\mathbf{f}_{EM} = d(\rho \cdot \mathbf{A}) / dt = \mathbf{A} \cdot \partial \rho / \partial t + \rho \cdot \partial \mathbf{A} / \partial t$  (A15.3)  
 With re-formulated unit equation for last term:  $\text{Vs}/\text{m} \cdot \text{As}/\text{m}^3 / \text{s} = \text{As}/\text{m}^3 \cdot 1/\text{m} \cdot \text{V} \rightarrow \rho \cdot \nabla \varphi$  (A15.4)  
 b2) Result in electrodynamics is Lorentz’ force law  $\mathbf{f}_{EM} = d(\rho \cdot \mathbf{A}) / dt = - \rho \cdot \nabla \varphi - \rho \cdot \partial \mathbf{A} / \partial t$  (A15.5)
- Q16) What is the relation between interdisciplinary force  $\mathbf{F}$  and (potential) energy  $W$ ?**  
 From  $N = \text{“?”} \cdot \text{Nm}$  or  $\text{“?”} = 1 / \text{m} \rightarrow$  space operator  $\nabla$ , with  $\mathbf{P} =$  pulse,  $W =$  energy, follows: (A16.1)  
 d’Alembert’s force  $\mathbf{F} = - \nabla W = - dW / dr = -dW / dt \cdot dt / dr = d\mathbf{P} / dt \rightarrow d(\mathbf{P} \cdot \mathbf{v} + W_{\text{pot}}) / dt = 0$  (A16.2)  
 Integration yields the non-relativistic energy law in classic physics:  $W_{\text{total}} = W_{\text{kin}} + W_{\text{pot}} = \text{const}$  (A16.3)
- Q17) a) What is the relation between energy  $W$  of an electromagnetic wave and frequency  $\nu$ ?**  
 Equivalent unit equation:  $\text{Ws} = \text{“?”} \cdot 1/\text{s}$  or  $\text{“?”} = \text{Ws} \cdot \text{s} \rightarrow$  Planck’s constant  $h$  (or  $\hbar$ ) (A17.1)  
 Result is Planck’s energy relation:  $W = h \cdot \nu$  or  $W = \hbar \cdot \omega$  or complex  $\rightarrow \underline{W} = \mathbf{i} \cdot \hbar \cdot \partial / \partial t$  (A17.2)  
 b) What is the relation between pulse  $\mathbf{P}$  of an electromagnetic wave and its wave length  $\lambda$ ?  
 Equivalent unit equation:  $\text{Ns} = \text{Nm} \cdot \text{s} \cdot 1/\text{m} = \text{Ws} \cdot \text{s} \cdot 1/\text{m}$  with wave vector  $\mathbf{k}$  yields both the (A17.3)  
 $\rightarrow$  de Broglie’s pulse:  $\mathbf{P} = h / \lambda$  or  $\mathbf{P} = \hbar \cdot \mathbf{k}$  or complex with  $\mathbf{i} = \sqrt{-1} \rightarrow \underline{\mathbf{P}} = - \mathbf{i} \cdot \hbar \cdot \nabla$  (A17.4)  
 $\rightarrow$  de Broglie’s wave equation:  $\Psi(\mathbf{r}, t) = \Psi_0 \cdot \exp[\mathbf{i} \cdot (\omega \cdot t - \mathbf{k} \cdot \mathbf{r})] = \Psi_0 \cdot \exp[\mathbf{i} / \hbar \cdot (W \cdot t - \mathbf{P} \cdot \mathbf{k})]$  (A17.5)  
 c) Can we derive the relation for the uncertainty principle in quantum mechanics where neither energy  $W$  & time  $t$  nor pulse  $\mathbf{P}$  & place  $\mathbf{r}$  can be exactly determined at the same time?  
 From (A17.2) & (A17.4) we get Planck’s constant  $h = W / \nu = \mathbf{P} \cdot \lambda$ , where  $\nu \sim 1 / t$  &  $\lambda \sim r$ . (A17.6)  
 and directly in “small  $\Delta$  - view”  $\rightarrow$  Heisenberg’s uncertainty relation  $h \approx \Delta \mathbf{P} \cdot \Delta \mathbf{r} \approx \Delta W \cdot \Delta t$  (A17.7)  
 d) Inserting (A17.2) & (A17.4) in (A16.3) with mass  $M$  and kinetic energy  $W_{\text{kin}} = M \cdot \mathbf{v}^2 / 2$  (A17.8)  
 $\rightarrow$  non-relativistic Schrödinger’s equation:  $\hbar^2 / (2 \cdot M) + W_{\text{pot}} = \mathbf{i} \cdot \hbar \cdot \partial / \partial t$  [applied on  $\Psi(\mathbf{r}, t)$ ] (A17.9)
- Q18) How can we easily derive a formula for arbitrarily formed current loops?**  
 Considering the electric Gauss’ law we quickly derive  $d\mathbf{D} = dQ / (4\pi r^2) \cdot \mathbf{e}_r$ , producing a (A18.1)  
 magnetic field  $d\mathbf{H}$  by moved charges  $\mathbf{v} \cdot dQ$  in units:  $\text{m}/\text{s} \cdot \text{As} = \text{A} \cdot \text{m} \rightarrow \mathbf{v}_{\text{dl}} \cdot dQ = \mathbf{I} \cdot d\mathbf{l}$  (A18.2)  
 With unit equation:  $\text{m}/\text{s} \cdot \text{As}/\text{m}^2 = \text{A}/\text{m}$  the transformation from  $d\mathbf{D}$  to  $d\mathbf{H}$  is  $\mathbf{v}_{\text{dl}} \times d\mathbf{D}$  (= A4.3) (A18.3)  
 The result is Biot-Savart’s law:  $d\mathbf{H} = \mathbf{v}_{\text{dl}} \times d\mathbf{D} = \mathbf{v}_{\text{dl}} \cdot dQ / (4\pi r^2) \times \mathbf{e}_r = \mathbf{I} / 4\pi \cdot (d\mathbf{l} \times \mathbf{e}_r) / r^2$  (A18.4)
- Q19) What is the difference between total derivative “ $d / dt$ ” and partial derivative “ $\partial / \partial t$ ”?**  
 i.e.  $d\mathbf{B}(t, \mathbf{r}) / dt = \partial \mathbf{B} / \partial t + \partial \mathbf{B} / \partial \mathbf{r} \cdot \partial \mathbf{r} / \partial t = \partial \mathbf{B} / \partial t + \partial \mathbf{B} / \partial \mathbf{r} \cdot \mathbf{v} = \partial \mathbf{B} / \partial t + (\mathbf{v} \nabla) \mathbf{B}$  (A19.1)  
 Basis for viewing point of moved systems  $\rightarrow$  Einstein’s special relativity theory (ref. Q20) (A19.2)  
 Vector gradient  $(\mathbf{v} \nabla) \mathbf{B} = d\mathbf{B} / dt - \partial \mathbf{B} / \partial t$  implies additional fields in Lorentz’ transformation (A19.3)  
 Applying unit checks i.e.  $\mathbf{B} = f$  (temperature  $T$ ) yields  $\rightarrow$  additional fields  $\partial \mathbf{B} / \partial T \cdot \partial T / \partial t$  etc (A19.4)  
 Most simple transformation from i.e. Faraday’s law:  $- d\mathbf{B}(\mathbf{v}) / dt = \text{curl } \mathbf{E}' = \text{curl}(\mathbf{E} + \mathbf{v} \times \mathbf{B})$  (A19.5)
- Q20) Which unit re-design leads from classic physics to Einstein’s etc most famous formulas?**  
 Newton’s force law in mechanics  $\mathbf{f}_M = d(\rho_M \cdot \mathbf{v}) / dt$  (ref. Q15a)  $\rightarrow \mathbf{F}_M = d(M \cdot \mathbf{v}) / dt$  ( $M$  is mass) (A20.1)  
 Equivalent unit equation:  $1/\text{s} \cdot \text{kg} \cdot \text{m}/\text{s} = \text{kg} \cdot \text{m}/\text{s}^2 = \text{N} = \text{Nm} / \text{m} = \text{Ws} / \text{m} \rightarrow \text{Ws} = \text{kg} \cdot \text{m}^2/\text{s}^2$  (A20.2)  
 The result is Einstein’s equation:  $\underline{E = M \cdot c^2}$  equivalence of energy  $E$  and mass  $M$ , where (A20.3)  
 the velocity unit  $\text{m}^2/\text{s}^2$  means the square of the speed of light  $c = 1 / \sqrt{(\mu_0 \cdot \varepsilon_0)} = \text{const}$ . (A20.4)  
 Viewing ( $\perp$ ) moved body ( $\Delta t'$ ,  $\mathbf{v}$ ), time in rest  $\Delta t \rightarrow$  Pythagoras yields:  $(c \cdot \Delta t')^2 = (\mathbf{v} \cdot \Delta t')^2 + (c \cdot \Delta t)^2$  (A20.5)  
 or Einstein’s time dilatation:  $\Delta t' = \Delta t / \sqrt{1 - (\mathbf{v}/c)^2}$ . Extending (A20.5) by unit checks to the (A20.6)  
 relativistic energy equation:  $E^2 = W_{\text{tot}}^2 = W_{\text{kin}}^2 + W_{\text{pot}}^2$ , with pulse  $\mathbf{P} = M \cdot \mathbf{v}$  & mass in rest  $M_0$  (A20.7)  
 $(M \cdot c^2)^2 = (\mathbf{P} \cdot c)^2 + (M_0 \cdot c^2)^2$  or Lorentz-Einstein’s mass equation:  $M = M_0 / \sqrt{1 - (\mathbf{v}/c)^2}$  (A20.8)  
 Complex notation of (A20.8) using (A17.2) & (A17.4) leads to Klein-Gordon’s equation or  
 $\rightarrow$  relativistic Schrödinger’s equation:  $\square \equiv \nabla^2 - 1/c^2 \cdot \partial^2 / \partial t^2 = (M_0 \cdot c / \hbar)^2$  (“space-time operator”) (A20.9)

## 2.2.6 Electrodynamics compared with other physical disciplines

Central physical disciplines are compared with electrodynamics neglecting Maxwell's  $d\mathbf{D}/dt$  – term in eq. (14) - (16), (18a), (19a), (20a) and considering it in eq. (17) - (20).

Electrodynamics Maxwell etc	$\text{curl}(1/\mu)\text{curl}\mathbf{A} = \mathbf{J} - \text{grad}[\gamma \cdot \varphi \pm (1/\mu_i) \cdot M_i]$	$-\gamma \cdot \frac{\partial \mathbf{A}}{\partial t} +$	$\gamma \cdot \mathbf{v} \times \text{curl}\mathbf{A}$	(14)
Hydrodynamics Navier-Stokes etc	$\text{curl}\eta\text{curl}\mathbf{v} = \mathbf{f} - \text{grad}[\rho + \rho_m \cdot \mathbf{v}^2/2]$	$-\rho_m \cdot \frac{\partial \mathbf{v}}{\partial t} +$	$\rho_m \cdot \mathbf{v} \times \text{curl}\mathbf{v}$	(15)
Thermodynamics Fourier-Helmholtz	$\text{div}\lambda\text{grad}T = Q - \text{grad}q_s$	$-c_p\rho_m \cdot \frac{\partial T}{\partial t} +$	$c_p\rho_m \cdot \mathbf{v} \cdot \text{grad}T$	(16)
Elastodynamics Newton-Euler, Lagrange etc	$\mu_m \cdot \Delta \mathbf{u} - \rho_m \cdot \partial^2 \mathbf{u} / \partial t^2 = -\mathbf{f} - (\mu_m + \lambda_m) \cdot \text{grad} \text{div} \mathbf{u} - \text{div} \boldsymbol{\sigma}$			(17)
	$\Delta \mathbf{u} - (\rho_m / \mu_m) \cdot \partial^2 \mathbf{u} / \partial t^2 = -1/\mu_m \cdot \mathbf{f}$	(18)	Elastostatics	$\Delta \mathbf{u} = -1/\mu_m \cdot \mathbf{f}$
Electrodynamics Maxwell, Ampere, Faraday, Gauss etc	$\Delta \mathbf{A} - (1/c^2) \cdot \partial^2 \mathbf{A} / \partial t^2 = -\mu_0 \cdot \mathbf{J}$	(19)	Magnetostatics	$\Delta \mathbf{A} = -\mu_0 \cdot \mathbf{J}$
	$\Delta \varphi - (1/c^2) \cdot \partial^2 \varphi / \partial t^2 = -1/\epsilon_0 \cdot \rho$	(20)	Electrostatics	$\Delta \varphi = -1/\epsilon_0 \cdot \rho$

Fig. 6 Interdisciplinary vector analytical structure identities in physics [1], [4]

Analogous expressions can be derived for diffusion equation in chemistry, Newton's mechanics, optics and acoustics etc. In eq. (14) the curl  $\mathbf{M}_p$ -term is re-formulated as grad  $M_i - \Sigma$  term of  $\mathbf{M}_p$ -components in  $i = x, y$ . Eq. (15) is also central for applications in aerodynamics. All these field equations (14) - (16) in Cartesian 2-dimensional coordinates will show identical structure, refer to Fig. 7. The central equation in non-linear elastodynamics is given by eq. (17), where  $\mu_m$  and  $\lambda_m$  are Lamé characteristics for material elasticity,  $\boldsymbol{\sigma}$  for non-linear tensions,  $\mathbf{u}$  for mechanical displacement and  $\mathbf{f}$  for external forces. Assuming linearity ( $\text{div}\boldsymbol{\sigma}=0$ ) and incompressible media ( $\text{div}\mathbf{u}=0$ ), elastodynamics is based on wave equations (18) - (20) with identical structure.

## 2.2.7 Electrodynamics directly integrated with other physical disciplines

*Magnetohydrodynamics: Hydrodynamics + Electrodynamics + Thermodynamics*

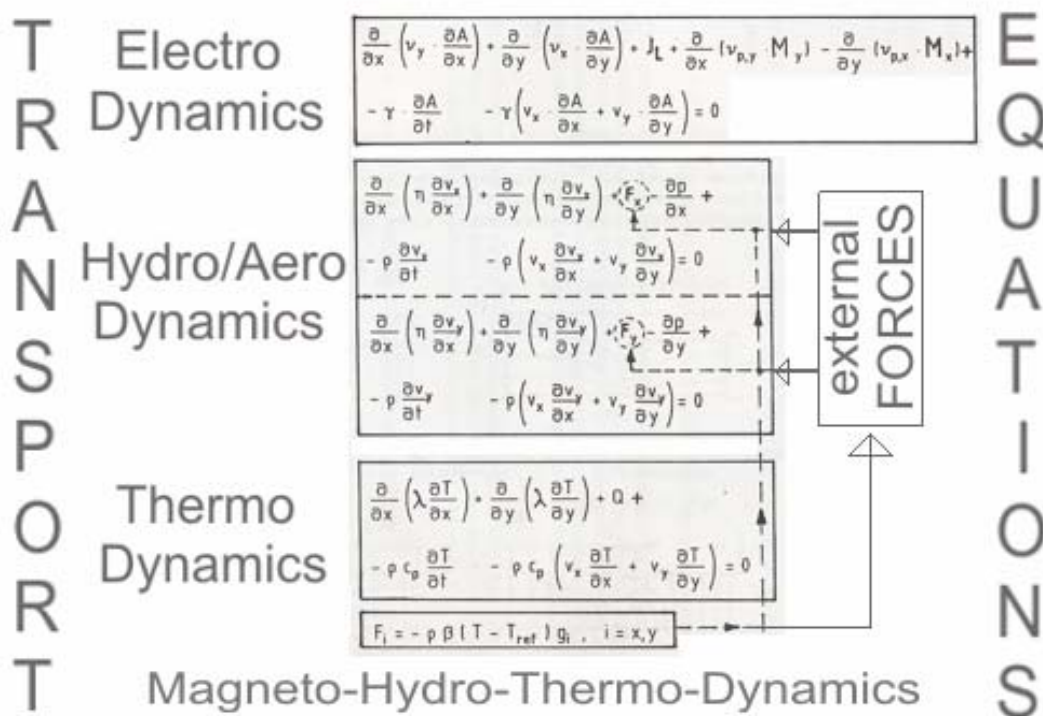


Fig 7. Interdisciplinary structure identities of different and hybrid physical disciplines [1], [11],

*Ferrohydrodynamic Bernoulli-Rosensweig equation based on magnetic fluids*

The Bernoulli equation can be deduced from Navier-Stokes equations and extended with a magnetic polarization  $\mathbf{M}_p$  to handle i.e. industrial separation of diamonds:

$$p + \rho_m \cdot \mathbf{v}^2 / 2 + \rho_m \cdot \mathbf{g} \cdot \mathbf{h} + \rho_m \cdot \int (\partial \mathbf{v} / \partial t) d\mathbf{l} - \int \mathbf{M}_p \cdot d\mathbf{H} = \text{const} \quad (21)$$

Eq. (21) describes i.e. lifting of "stones" in magnetic fluids by magnetic fields [1].

## Magnetostriction and Electrostriction: Elastomechanics + Electrodynamics + Entropy

The deformation force densities  $f_{MS}$  (or  $f_{ES}$ ) of ferromagnetic (or dielectric) materials with density  $\tau=1/v$  caused by magnetic (or electric) fields  $H$  (or  $E$ ) can be derived by means of entropy. The converse effect applying mechanical pressure  $p$  to certain non-conducting crystals producing electric charges is piezoelectricity. All effects may depend on temperature  $T$ , too. In Fig. 8 interdisciplinary derivations are shown.

Maxwell $\rightarrow$ Poynting $E \times H \rightarrow$ energy thermodynamics $\rightarrow$ "unavailable for work" entropy $dS = dQ/T$	(22)
Internal system energy $dW_i = f$ [specific volume $v(p), T, H$ (or $E$ )] = transported heat $dQ$ + total work $dW_w$	(23)
entropy $dS_{MS}$ with $dW_i = f(v, H, T)$ for magnetostriction	$dW_{w,MS} + p \cdot dV = H \cdot d(\mu \cdot H) = \mu \cdot H dH + H^2 \cdot [(\partial \mu / \partial v) dv + (\partial \mu / \partial T) dT]$ $dS_{MS} = dQ/T = \frac{1}{T} \left( \frac{\partial W_i}{\partial v} + \frac{p}{v} - H^2 \frac{\partial \mu}{\partial v} \right) dv + \frac{1}{T} \left( \frac{\partial W_i}{\partial H} - \mu \cdot H \right) dH + \frac{1}{T} \left( \frac{\partial W_i}{\partial T} - H^2 \frac{\partial \mu}{\partial T} \right) dT$
	from eq. (22) - (25) $\rightarrow$ force density $f_{MS} = - \text{grad } p(v, H, T)$ , neglecting $T$ : eq. (26) - (27)
Magnetostriction force density	$f_{MS} = \frac{1}{2} \text{grad} \left( H^2 \cdot \tau \cdot \frac{\partial \mu}{\partial \tau} \right)$ (26)
Electrostriction force density	$f_{ES} = \frac{1}{2} \text{grad} \left( E^2 \cdot \tau \cdot \frac{\partial \epsilon}{\partial \tau} \right)$ (27)

Fig 8. Interdisciplinary entropy equations for magnetostriction + electrostriction [6], [1]

### 3 Interdisciplinary industrial applications in mechatronics

Three developments in the huge field of motor car production, magnetic gripper design in robotics, motor car anti-vibration systems and computer hard disk drives will demonstrate actual industrial applications in mechatronics based on electrodynamics.

#### 3.1 Motor car production based on electrodynamics

In Fig. 9 we see a graphical overview of the actual topics of this publication concerning motor car production and influences. The field numerical optimization of the actual holding and stacking system in world wide motor car production is the special focus in this chapter 3.1.

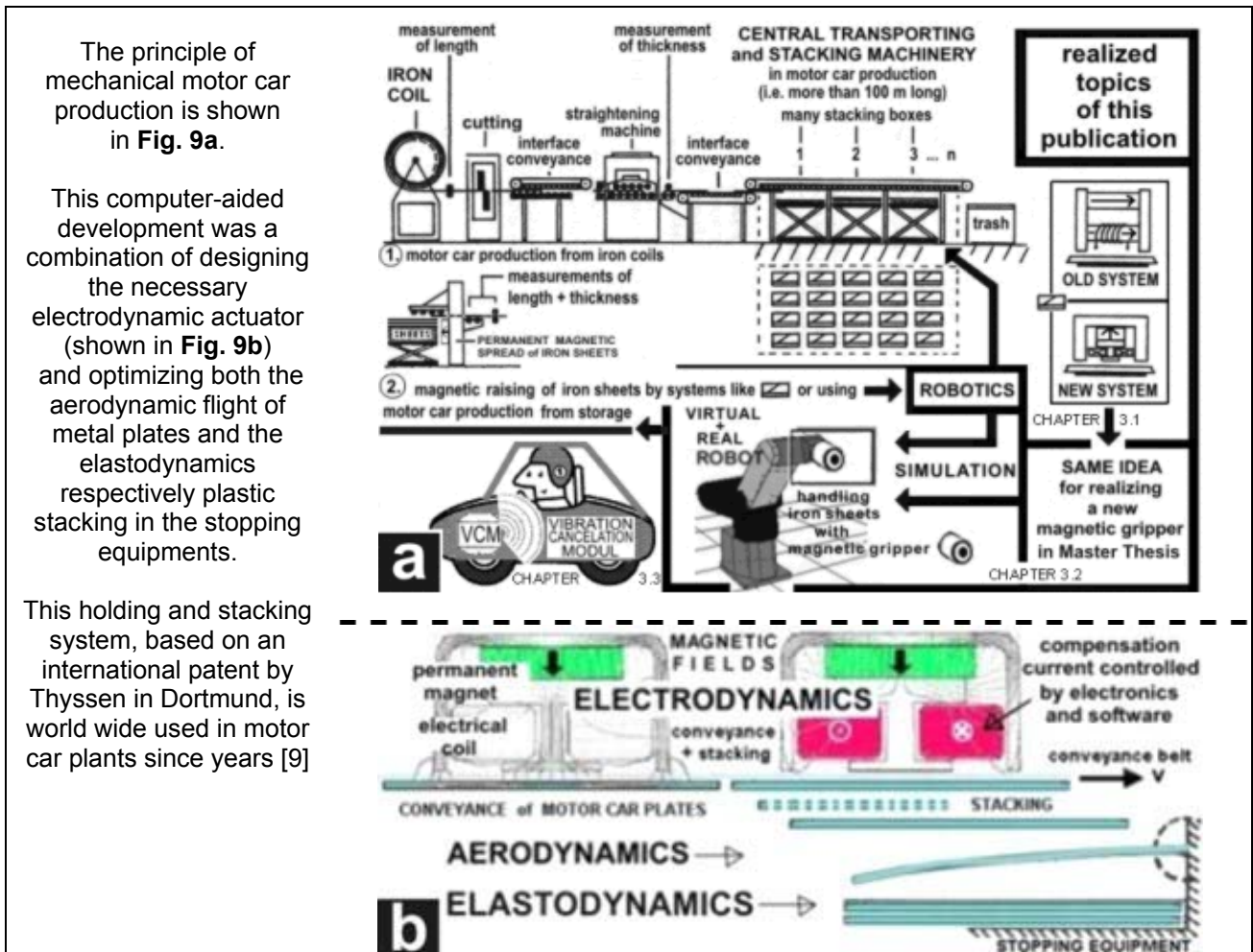


Fig 9. Mechanical motor car production based on electrodynamics [1], [10], [11],

### 3.2 Gripper design in robotics based on electrostatics

Based on the idea for the controlled actuator in motor car production, the following magnetic gripper was developed, simulated and realized in Fig. 10.

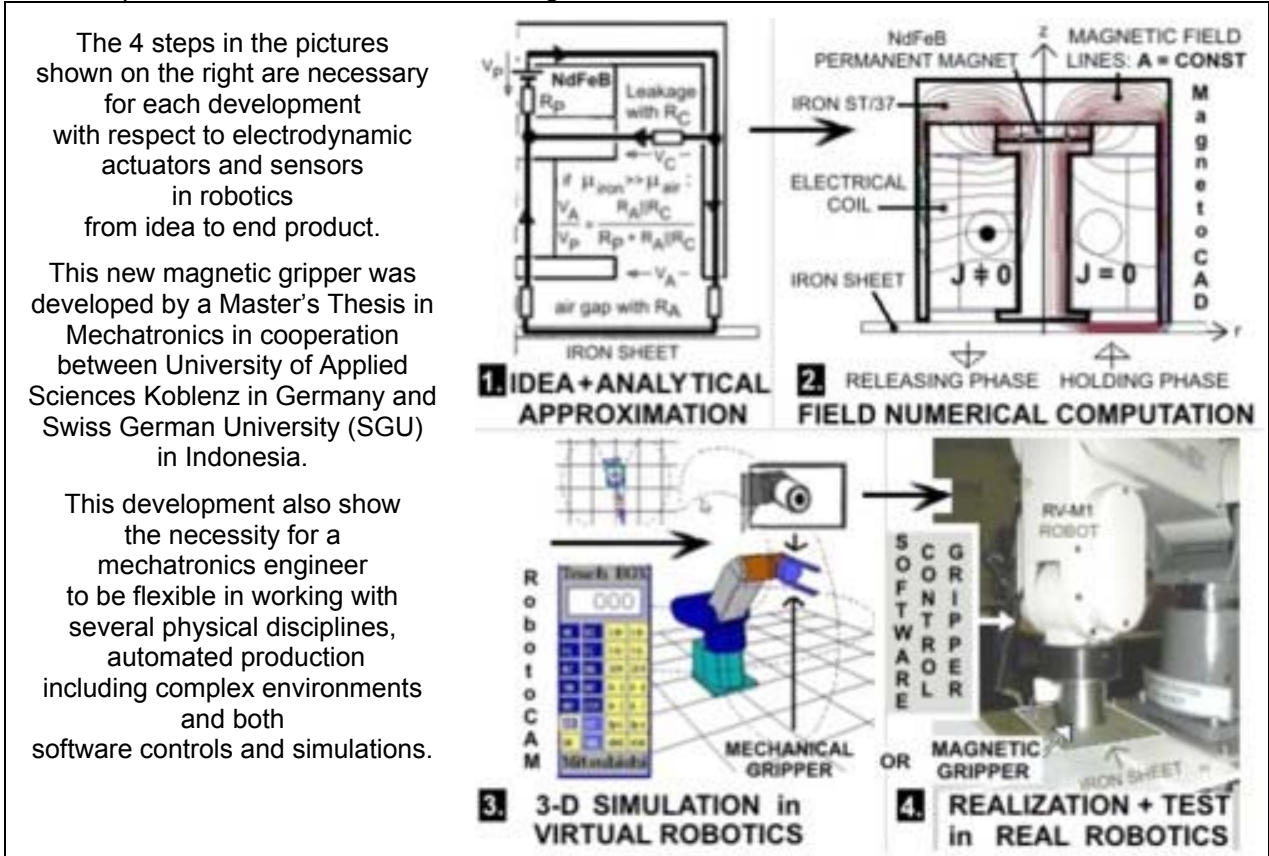


Fig 10. New magnetic gripper design for handling metal sheets in 4 steps [2], [11]

### 3.3 Motor car anti-vibration system based on electrostatics

The cancellation of noise inside motor cars, using software controlled actuators, is of great importance in all motor car plants. The design of such anti-vibration systems (VCM) in Fig. 11 involves several interdisciplinary areas in physics such as acoustics, electrostatics, thermodynamics, hydrodynamics, mechanics, elastodynamics and sound design, too.

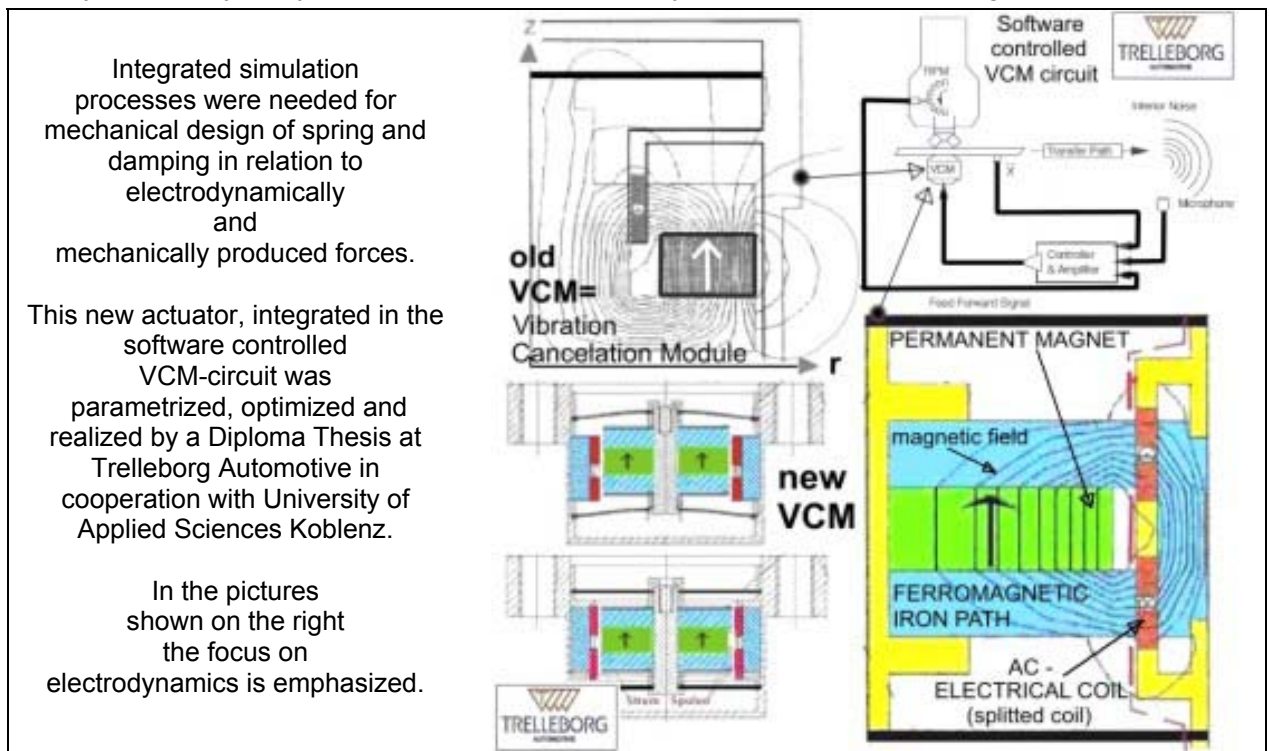


Fig 11. Motor car anti-vibration system based on electrostatics [3], [11]

### 3.4 Computer Hard Disk Drives

Complex concepts and applications of electrodynamics are the basis for a huge variety of Hard Disk Drives in computers (i.e. often like in Fig. 12 or other special variants). Though very different in construction details, all hard disk drives are consisting of electrical coils, permanent magnets, iron parts and often additional copper plates or closed coils in form of a “shorted turn” (ref. to Fig. 13 and 14, the principle of a Winchester-Hard-Disk-Drive). The main task of these drives is to perform and to control the accelerated movements of magnetic heads for an exact and fast reading and writing of data on the magnetic hard disk.

Combined analytic modelling and computer aided simulation of mechanical and electromagnetic devices in mechatronics is necessary to solve and to simulate the behaviour computer hard disk drives, i.e. Winchester drives.

For analytical calculation of electromagnetic fields in mechatronic systems and interdisciplinary analogies: Thinking in magnetics with concentrated field elements (R, L and often C) and solving dynamics by well known electrical circuit methods (i.e. with MATLAB, Simulink aided by FEMLAB, MAXWELL & MagnetoCAD)

The analytical equations (I – IV) are shown below considering No “Shorted turn” (index “nS”)

Modelling the influence of the “Shorted turn” (index “S”) as a transformer with “1” turn on the secondary side we can use the equations (V + VI)



Fig 12. Often constructed hard disk drive (photo)

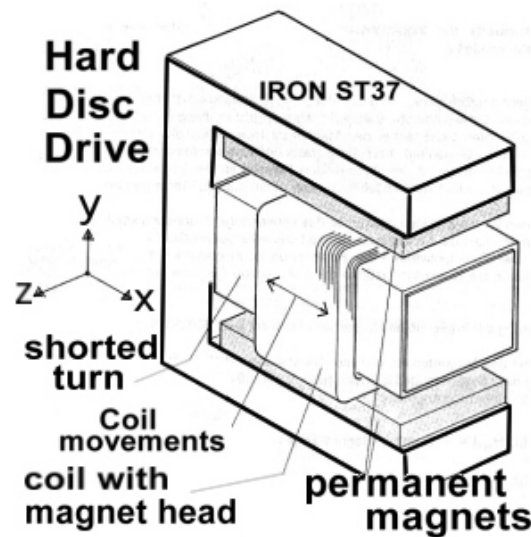


Fig 13. Principle of Winchester hard disk drive [1]

$$\begin{aligned}
 u_{1,nS} &= R_1 \cdot i_1 + d(L_1 \cdot i_1) / dt + u_{ind} \quad \text{(I)} \\
 u_{ind} &= k_1 \cdot \omega \quad \text{(II)} \\
 d(J_{mec} \cdot \omega) / dt &= T_M - T_W \quad \text{(III)} \\
 T_M &= k_2 \cdot i_1 \quad \text{(IV)} \\
 u_{1,S} &= u_{1,nS} + d(M \cdot i_S) / dt \quad \text{(V)} \\
 0 &= R_S \cdot i_S + d(L_S \cdot i_S) / dt + d(M \cdot i_1) / dt \quad \text{(VI)}
 \end{aligned}$$

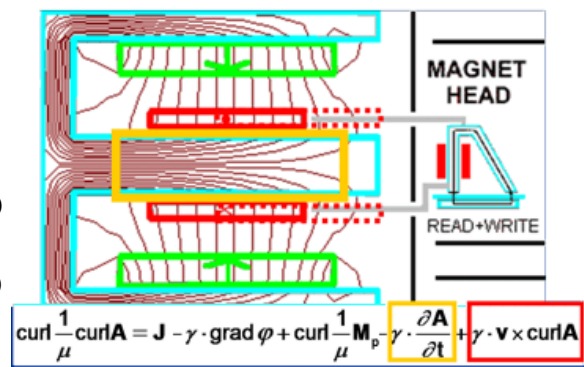


Fig 14. Winchester hard disk drive with magnetic field values for eq. (I-VI) and eddy current equation from Maxwell

If inductances  $L_1$ ,  $L_S$ , mutual inductance  $M$  and mass moment of inertia  $J_{mec}$  are constant eqs. (I-VI) can easily be simplified: i.e.  $d(L \cdot i) / dt = L di / dt + i dL / dt$ , where the last term is zero. MATLAB and Simulink are mighty systems for simulating problems in mechatronics. But without the numerical computation of central electromagnetic field values, primarily  $L$  and  $M$ , analytical simulations may yield false results not usable for optimized applications in practice. Directly from equations (I-VI) we can sketch the block diagram and the automation graph.

## 4. Conclusion

The high aim of optimizing the integration of mechanical engineering, electrical engineering and information technology in mechatronics can often be reached by preferred usage of advanced field theory in electrodynamics. Working with extended Maxwell's equations electrodynamics in mechatronics often leads to new developments and interdisciplinary influences are easier and faster to approximate. Quick derivation of interdisciplinary & complicated equations in physics can be achieved by using extremely helpful and mighty unit checks. Furthermore other electrodynamic influences especially caused by external waves and fields with respect to electromagnetic compatibility problems can be handled and corrected. The focus on four described developments such as motor car production, magnetic gripper design in robotics, motor car anti-vibration systems and computer hard disc drives show the necessity for a mechatronics engineer to be flexible in working with several physical disciplines (refer to right shown Mind Map with Memo Maps in Fig. 15), highly automated production including complex environments and both a variety of different software controls and simulations

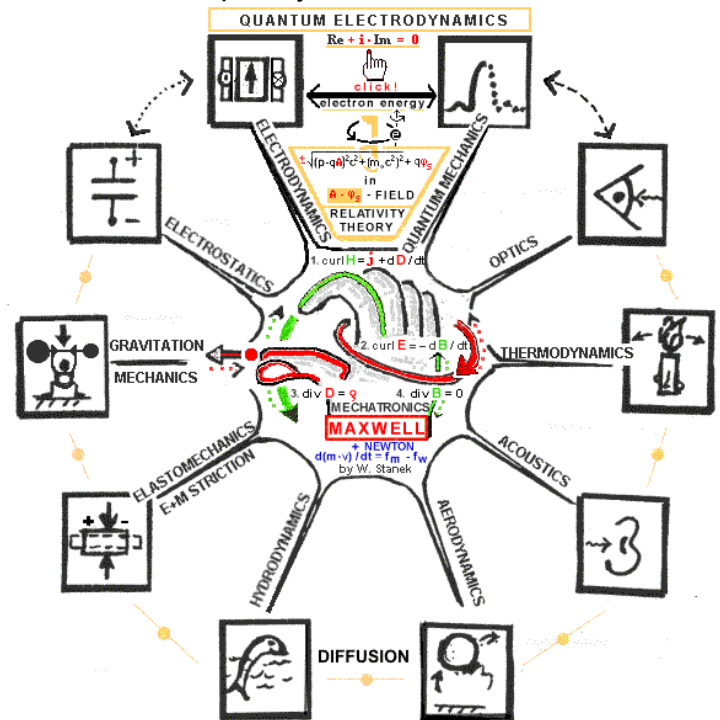


Fig 15. Interdisciplinary analogies in mechatronics based on Maxwell's equations [1], [11]

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